

Some Characteristics of Lucas's *ASL* & *LAS* under Generalized Conditions

— A Synthetic Understanding of
Both Lucasian and Keynesian Systems —

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I.

As well known for most economists notably, Lucas's model of aggregate supply equation is characteristically shown as follows (probable error term is omitted).¹⁾

$$Y_t - \bar{Y} = a \cdot (P_t - P_t^*) \quad (\text{N.B. } a > 0)$$

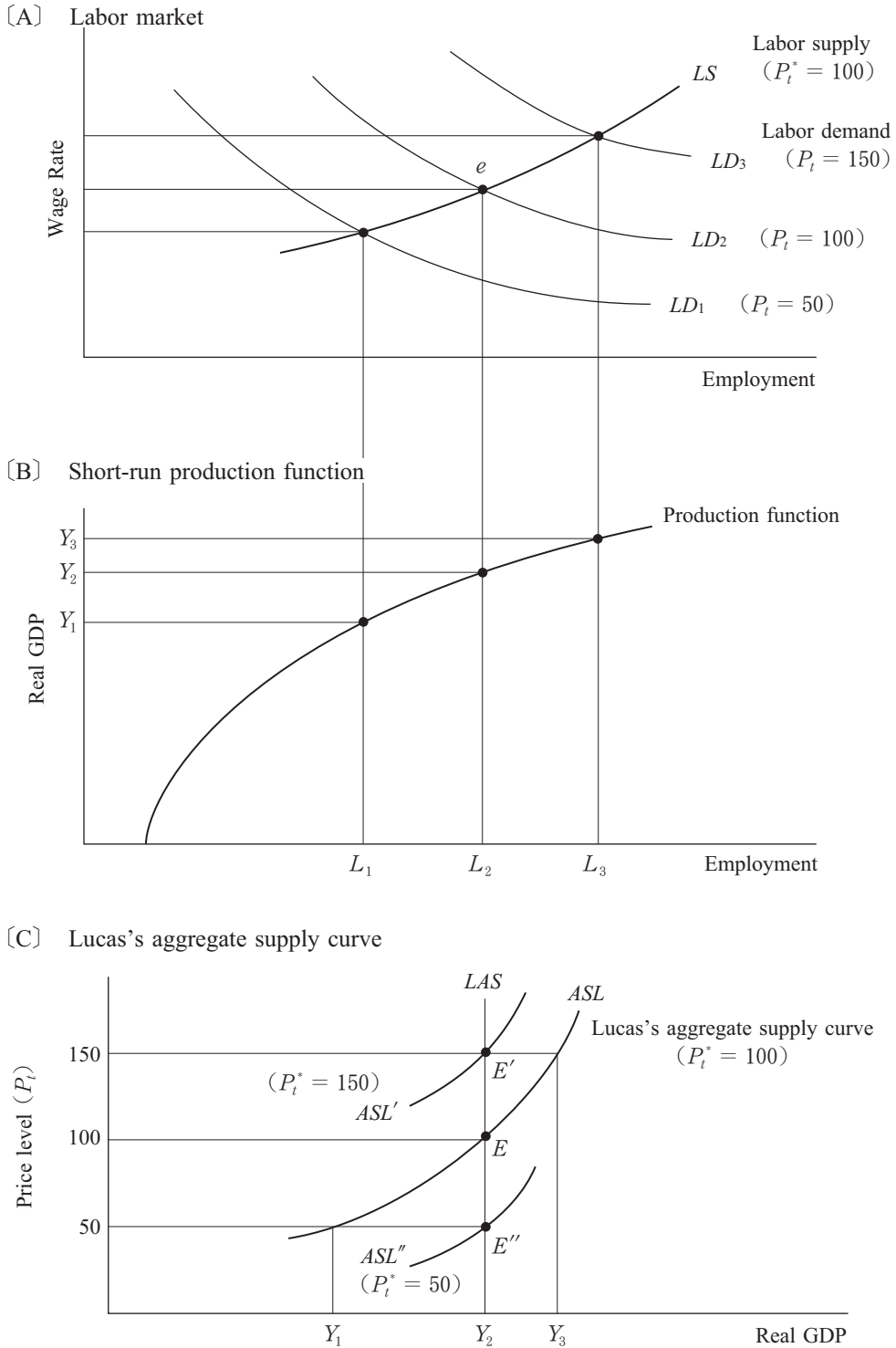
[Notation]

Y = current GDP \bar{Y} = GDP in "natural unemployment rate" situation
 P = actual price level P^* = expected price level
 t = sub-affix referring to time

Due to the equation, in case, as the result of the equilibrium state of things in labor market, the actual price level for enterprises' products is coincide with the level of rationally expected price of labor suppliers, i.e., $P_t = P_t^*$, then the production level of Y_t could no longer deem to rise from the certain level of \bar{Y} . Even in the equilibrium as $P_t = P_t^*$, it does not necessarily mean full employment. That is to say, \bar{Y} is the definite certain GDP level in "the natural unemployment rate" situation for the economy. Merely in non-equilibrium case as $P_t > P_t^*$, current GDP could be increased as $Y_t > \bar{Y}$. This is just the widely known Lucasian theorem.

In Graph 1 the theorem of Lucasian model is illustrated through the logic of its inference.²⁾

Graph 1 Derivation of the Lucas's Aggregate Supply Curve (ASL & LAS)



Cf. M. Parkin & R. Bade, *Macroeconomics*, 2nd edition, Prentice Hall, Inc., Englewood, New Jersey, 1992, pp. 596-599.

Graph 1 [A] exhibits some supply and demand functions (curves) in labor market. Labor supply function, LS , is based on P_t^* . While, any labor demand function LD_1 , LD_2 , or LD_3 is based on each P_t . Only the cross point of both curves, e , at where $P_t = P_t^*$, means the equilibrium situation in the labor market.

Graph 1 [B] shows a short-run production function of “decreasing marginal-return type”. Needless to say, it is identical with “increasing marginal-cost type”. Accordingly, on conditions of competitive market economy, P_t would be justly no more than the remunerative price level for enterprises' products.

In this production function, only the independent determinant is “employment”, that is labor input. Consequently, the input of another factor of production, i.e., the level of fixed-capital operation, is implicitly assumed to be given as an immutable constant. Therefore, the content of such remunerative price for enterprises' products, P_t , is nothing less than the marginal cost of labor input, i.e., which is exclusively determined through the marginal wage rate. Accordingly, the equilibrium condition on the production function should be corresponded to the point e in Graph 1 [A] which is the equilibrium point of labor market. Consequently, the equilibrium level of real GDP is Y_2 in Graph 1 [B]. This causality is the very “Lucas's aggregate supply curve”, *ASL*, as shown in Graph 1 [C].

As illustratively shown with Lucas's *ASL* curve in Graphs 1 [C], in case real GDP increases from Y_1 to Y_2 , it would necessarily to be accompanied with a rise of P_t from 50 to 100, due to the augmented marginal cost. Similarly, an increase in real GDP from Y_2 to Y_3 , corresponding to some increments of demand (e.g. induced by Keynesian type fiscal policy), must go with the rise of P_t from 100 to 150. The increases of P_t would inevitably be followed by similar rises of the level of P_t^* and wage rates through rational expectation activities of people, accompanying the likewise shifts of labor supply curve, LS . Needless to say, its consequences should be the upward vertical shifts of the production function by same rates. However, the shifts of production function would by any means never be “shifts in real term” but only “shifts in nominal term”. Therefore, here is only the vertically paralleled shifts of *ASL* curves, as $ASL'' \rightarrow ASL \rightarrow ASL'$ cited in Graph 1 [C]. Then we can see a vertical locus line, *LAS*, of the equilibrium points as $E'' \rightarrow E \rightarrow E'$.

Eventually, the equilibrium level of real GDP, Y_2 , is immutable, while only the price level, P_t , has changed. This vertical *LAS* line is just the so-called “Lucasian long term aggregate supply curve”. Needless to say, the implication of *LAS* is nothing but a negative judgment against the efficacy of Keynesian policy thought. It should be noted here that the Lucasian *LAS'* logic would be carried

through even on the occasion where the labor supply curve, LS , is consistently shaped with upward slope, i.e., it has no any horizontally flat part as cited in Graph 1 [A].

In abbreviating explanation, the Graph 1 shows Y_2 itself as nothing else but being identical with \bar{Y} , i.e., the level of GDP for “natural unemployment rate” situation, in Lucasian equation model cited above.

However, here, it also should be noted that, in the Lucasian world as cited in Graph 1, the LAS' logic of negation against economic growth might be similarly applicable for all the growth based on purely private activities, under the surrounding conditions even without any Keynesian policy. In other wards, as far as the presumed existence of short-run production function as cited in Graph 1 [B] and the hypothesis of people's rational expectation on prices are given, the Lucasian pessimistic logic of LAS shown in Graph 1 [C] would be widely applicable to any sort of economic activities in market economy.

II.

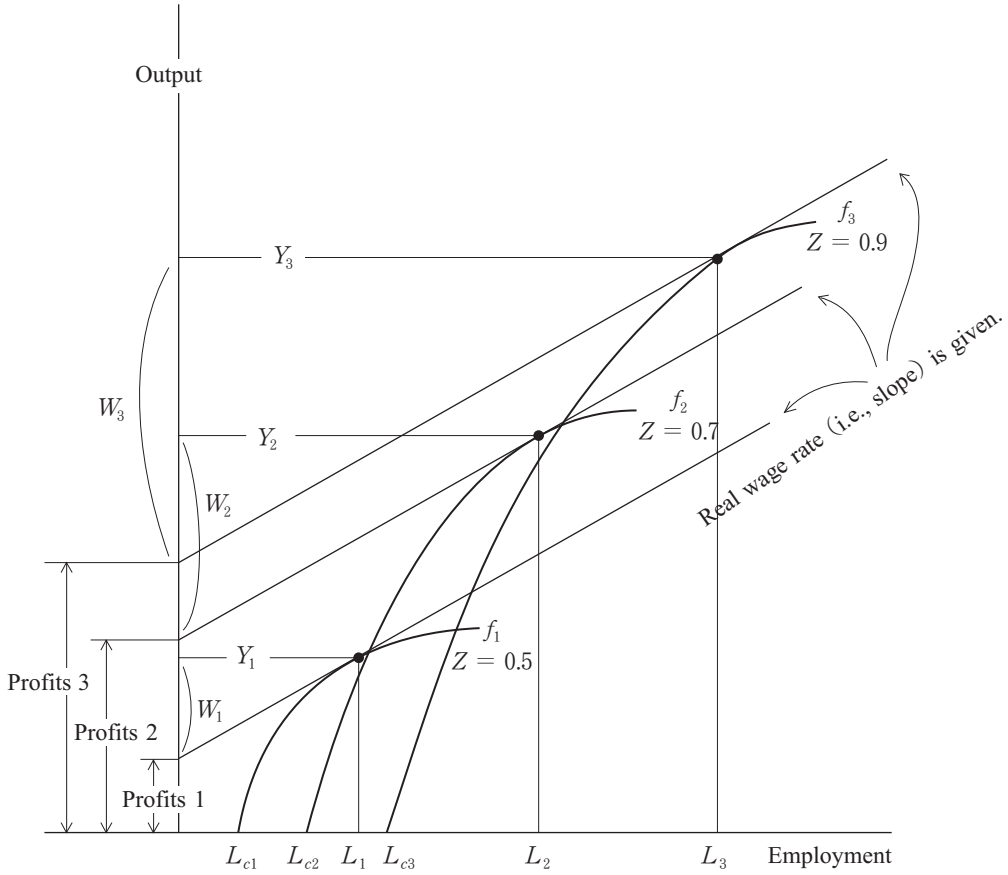
In the production function as shown in Graph 1 [B], the input-volume of capital operation is assumed to be a given constant postulate as cited above. That is to say, not only the macro-total of enterprises' fixed capital stocks but also its rate of operation is assumed to be as the exogenously given immovable conditions. Needless to mention, the Lucasian presumption is extremely far from reality.

Certainly, as a permissible simplification for theory, the macro-total volume of enterprises' fixed capital stocks might be assumed to be given as a constant for a “short-term” of three or four years, i.e., an approximation to “gestation period” of investment-engineering works. However, even during such a short-term period, the rate of operation of enterprises' fixed capital stocks surely would be changed, corresponding to some movings of total demand caused by transitional phases of trade cycle.

Naturally, the upward changes in the rate of operation of enterprises' fixed capital stocks corresponded with augmented demands might induce some shifts of production function in real terms, bringing about increases in productivity and profits. Such effects of changes in the rate of enterprises' fixed capital operation and the shifts of production function are cited in Graph 2.

The graph illustrated with Z of operation-rates of enterprises' fixed capital is exemplified key shifts' pattern of the short-run production function, f , in

Graph 2 Short-run Production Function and its Shifts



- f Production function
- Z Operation rate of fixed capital stock
- Y Output
- L Labor input (i.e., employment)
- L_c Fixed cost (in terms of labor input) inclusive of warming-up expenses
- W Compensations for labor

accordance with the changes of $Z = 0.5 \rightarrow Z = 0.7 \rightarrow Z = 0.9$ as well as $f_1 \rightarrow f_2 \rightarrow f_3$. Macroscopically speaking, the situation as $Z < 1.0$ means some existence of deflationary gap in the economy, even when the labor market has reached to the state of things of an equilibrium satisfying the necessary condition as $P_t = P_t^*$.³⁾ In the graph, symbol L_c as starting point for each f means the fixed cost (in terms of labor input) inclusive of warming-up expenses for idle plants. The position of L_c on the horizontal axis shifts rightward as $L_{c1} \rightarrow L_{c2} \rightarrow L_{c3}$. However the main characteristic of the shifts of production function is decisively upward shifts.

The most important meaning is that, as clearly shown in Graph 2, despite of the increases in real output as $Y_1 \rightarrow Y_2 \rightarrow Y_3$, there are no rises of marginal cost due to the real-term shifts of production function as $f_1 \rightarrow f_2 \rightarrow f_3$. In other words, the shifts of production function could free itself from the fetter of “decreasing marginal-return = increasing marginal-cost”.

In Graph 2 we can see the illustrated situation as Profits 3 > Profits 2 > Profits 1. Needless to say, this increases in profits are the necessary conditions for real-term shifts of production function as $f_1 \rightarrow f_2 \rightarrow f_3$.

Here in Graph 2, for the sake of simplifications, real wage rate is assumed as could to be a given constant (i.e., it would be a state of things on flat line part of labor supply curve). However, even in case, corresponding to augmented production, some rises in real wage rate come about due to the upward slope on labor supply curve, only so far as the necessary conditions regarding profits cited above are fulfilled, the shifting process of production function, more or less, might be caused.

The above stated realistic paradigm implied in Graph 2 would inevitably be a very important thought of generalization-adjustment for Lucas’s model.

III.

In Graph 3 [A], two labor supply functions (curves) commonly have a specific feature, i.e., each of both curves, LS and LS' , has a horizontally flat line connected with an upward slope part at the inflection point level of employment, e^* (which is corresponded to L^* in Graph [B]). It would be a fairly plausible feature of macro labor supply curve.

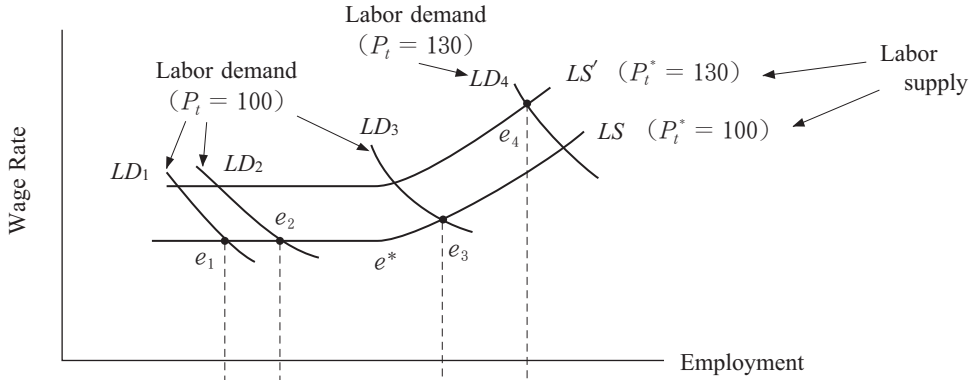
Here, the exemplified three labor demand curves, LD_1 , LD_2 and LD_3 are based on same level of actual remunerative prices for enterprises’ products, i.e., $P_t = 100$. Such stability of P_t would be attributable to the shifts (in real terms) of production function as cited in Graph 2 or Graph 3 [B].

The equilibrium points at the condition as $P_t = P_t^* = 100$ in labor market are e_1 , e_2 and e_3 as cited in this Graph 3 [A]. Corresponding to it, the equilibrium points on production function are E_1 , E_2 and E_3 . Then the equilibrium levels of real GDP are Y_1 , Y_2 and Y_3 as shown in Graph 3 [B].

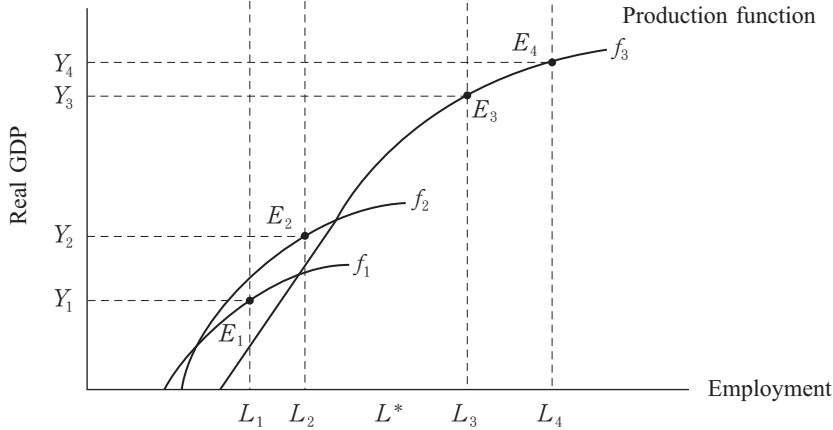
As shown in Graph 3 [C], it should be considered that an augmented total of effective demand could surely raise the level of real GDP as $Y_1 \rightarrow Y_2 \rightarrow Y_3$ at stable equilibrium level of prices as $P_t = P_t^* = 100$, even in case of analysis by using the Lucasian concept of macro equilibrium. This is meant that the slopes

Graph 3 Derivation of the Lucas's *ASL* & *LAS* in Cases of the Shifts of Short-run Production Function

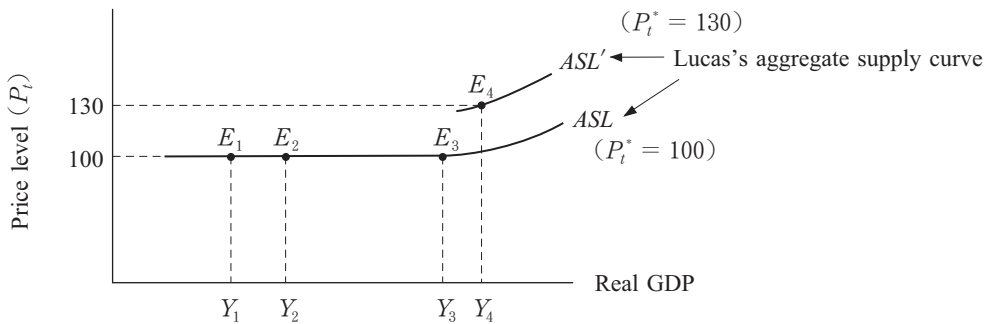
(A) Labor market



(B) Short-run production function



(C) Lucas's aggregate supply curve



of Phillips curve (in terms of the rate of price level's increase as an inverse proportion to the unemployment ratio) and Lucas's *LAS* are both could be flat.

More over, citing E_4 in Graph 3 (C), the price rise as $100 \rightarrow 130$ might

patiently be accepted, it would be a sufficient reward that the equilibrium level of real GDP could increase as $Y_3 \rightarrow Y_4$, through, say, some Keynesian fiscal policy.

As widely known, in ordinary form of the Phillips curve, unemployment rates are sized up on horizontal axis, while the scale for price level is vertical line. Therefore, the course of increase in real GDP (as $Y_3 \rightarrow Y_4$) followed by the rise of price level (as $100 \rightarrow 130$) is none other than the Phillips curve with an ‘downward’ sloping (namely, upward sloping in direct proportion to real GDP growth).

Here, the eminent assertion by Milton Friedman might be recalled.⁴⁾ As well known, his study has been derived from the supposition of ‘downward’ sloping in Phillips curve (in terms as cited above). Using same symbols and suppositious numerals as cited above in this paper for convenience’ sake, the Friedman’s thesis would be as that, in case P_t^* come up to P_t , real GDP necessarily be come down from Y_4 to Y_3 again, while the risen level of prices staying as it is as 130.

However, this Friedman’s view could not be admissible. Because, both Y_3 and Y_4 are quite perfectly stable equilibrium values of real GDP in terms of Lucasian concept and the real total volume of enterprises’ profits in Y_4 are necessarily larger than it in Y_3 . In essence, Phillips curve includes any sort of *ex-post* effects of the shifts of production functions. Consequently, it never could be identical with the Lucasian *ASL* or the usual *ex-ante* supply curve.

In sum, Graph 3 [C] shows nothing but the logical conclusion of the adjusted-generalized Lucasian model which has been enunciated in the paper.

The conceptual meaning of each increased equilibrium GDP in real terms, i.e., Y_1 , Y_2 , Y_3 , and Y_4 , is identical with the meaning of \bar{Y} in the Lucas’s model of aggregate supply equation cited at the opening paragraph of the text above. However, while the value of such original Lucasian \bar{Y} is ratiocinated as an immutable constant value corresponded to some “natural unemployment rate” situation of the economy, on the contrary to it, in the adjusted-generalized model, the equilibrium GDP in real terms could be increased for $Y_1 \rightarrow Y_2 \rightarrow Y_3 \rightarrow Y_4$ due to the shifts’ effects of short-run production function as shown in Graph 3 [B].

Therefore, as exemplified in Graph 3, in case of situations brought about through the shifts of short-run production function, the value of “natural unemployment rate” would be inevitably and automatically adjusted to downward direction, correlating to the increases in GDP (i.e., $Y_1 \rightarrow Y_2 \rightarrow Y_3 \rightarrow Y_4$) and in employment (i.e., $L_1 \rightarrow L_2 \rightarrow L_3 \rightarrow L_4$). That is to say, under the conditions implied in the situation as shown in Graph 3, so-called “natural unemployment rate” is not a definite constant value now, but rather flexibly changeable variable.

Its meaning is that the concerned economic system has a possibility for approach to full-employment by the use of some Keynesian fiscal policy.

In short, the most important finding in this study is that the adjusted-generalized Lucasian model would be consistent with the most Keynesian policies. In other words, Graph 3 in the text especially suggests a sort of synthetic understanding for both Keynesian and Lucasian systems. The implications of it would be very important for any drafting of fiscal policies.

Notes

- 1) Cf., R. E. Lucas Jr., (1973) "Some International Evidence on Output-Inflation Trade-offs", *The American Economic Review*, Vol. 63, No. 3, pp. 327–328, R. Maddock & M. Catter, (1982) "A Child's Guide to Rational Expectations", *Journal of Economic Literature*, Vol. 20, pp. 39–51, Appendix B.
See also, H. Yoshikawa, (1984) *Research on Macroeconomics* (Tokyo University Press, in Japanese), pp. 60–63, and T. Shizuki & Y. Mutoh, (1981) *Rational Expectation and Monetarism* (Nihon Keizai Shinbunsha, Tokyo, in Japanese), p. 98.
- 2) The graph basically follows the widely used textbook, M. Parkin & R. Bade, (1992) *Macroeconomics*, 2nd ed. (Prentice-Hall, Inc., Englewood Cliffs, New Jersey), pp. 596–599.
- 3) Through past a few decades, Japanese economy has suffered from the long existent cumulative occurrence of huge deflationary gap. See, H. Niwa, (2000) "The Recent Deflationary Gap in Japan; A Quantitative Measurement", *Journal of Asian Economics*, Vol. 11, No. 2, pp. 245–258, and also see H. Niwa, (2003) "Deflationary Gap in Japan, 1970–2000: A Quantitative Measurement", *Journal of Economic Policy Studies*, Vol. 1, No. 1 • 2, pp. 79–101.
Cf., H. Niwa, (2006) *The Foundations of New-Orthodox Keynesian Policy Thought* (Gakujutsu-shuppankai, Tokyo, in Japanese), pp. 157–225.
- 4) Cf., M. Friedman, (1977) "Nobel Lecture: Inflation and Unemployment", *Journal of Political Economy*, Vol. 85, No. 3, pp. 451–472.

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